Weaving a Uniformly Thick Sheet from Rectangles

Eli Davis Erik D. Demaine Martin L. Demaine Jennifer Ramseyer

Masscahusetts Institute of Technology

Any child knows how to weave a few strips of paper into a sheet, which happens to be of uniform thickness. However, the size of the sheet in this simple weaving is limited by the length of the strips used in creation. This sheet also requires some sort of external locking mechanism to hold it together. Without tape or a non-uniform edge, this sheet will slide apart.

By contrast, we show how to weave together finite-length strips into an infinite sheet of uniform thickness. In addition, our sheet is locked, and will not slip. However, our weaving requires more layers than the child's model. For an arbitrary rectangular strip, the sheet is, at worst, eight layers thick. For an arbitrary polygonal "strip", the sheet is no thicker than eighteen layers. In the worst case, we conjecture that these are also the lower bounds. In some special cases, we show that the sheet can be thinner; for example, using 1×5 rectangles, we can make a sheet that is just five layers thick (see Figure 1). It is also possible to reduce to four layers of thickness with carefully designed concave polygons. This paper proves our upper bounds on thickness, and gives general sheet-weaving algorithms.

While creating a locked, infinite sheet of uniform thickness proves to be relatively simple, creating a finite sheet is far more complex. In a finite sheet, boundary conditions come into play. If we just apply a portion of our infinite constructions, the edges of the sheet are ragged, and locking the sheet while keeping the thickness constant proves tricky. We describe solutions to these problems, resulting in locked, uniform-thickness, finite woven sheets, but at the price of further increased thickness.

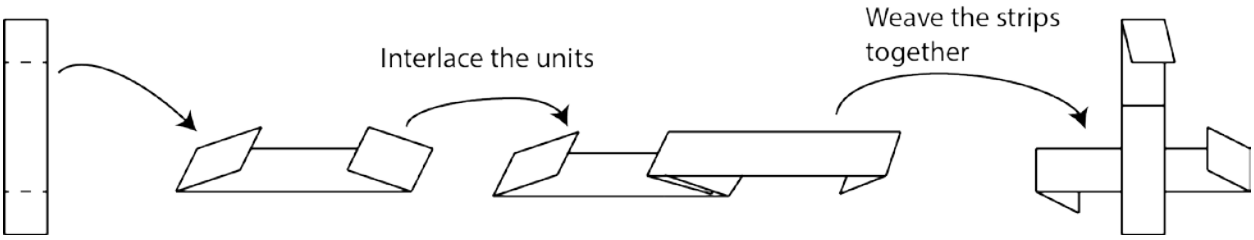

Figure 1: Creating a 5-layer woven sheet from a 1-by-5 rectangle.