# A REVIEW ON CURVED CREASES IN ART, DESIGN AND MATHEMATICS

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**Abstract***: In this paper we review masterpieces of curved crease folding, the deployed design methods and geometric studies regarding this special kind of paper folding. Our goal is to make this work and its techniques accessible to enable further development. By exploring significant works of the past and present of this still underexplored field, this paper aims to contribute to the history of this geometry and the development of novel design methods and a brief survey of approaches in geometry .*

**Keywords**: origami, curved creases; paperfolding; digital fabrication.

## 1. INTRODUCTION

Traditional paper folding mostly uses straight creases. We call this type of origami prismatic origami, since straight creases surround planar facets and compose a polyhedral surface (e.g., PCCP shells and Miura-ori (Miura, 1970) and Resch's structure (Resch et al., 1970)). Here, by altering the crease and making it into a curved folding, the surface suddenly becomes a complex three-dimensional form that cannot be described easily by simple parameters as vertex coordinates. Curved folding is a hybrid of folding and bending a sheet, and the surface is comprised of curved creases and smooth developable surface patches. This can be compared to prismatic origami being the result of pure folding, and the smooth developable surface created from pure bending of a sheet.

The hybrid property of curved folding has an advantage when used to form a 3D surface from sheet materials. When we try to form a surface by pure bending, the shape is limited to simple geometries such as cones, cylinders and tangent surfaces. Prismatic origami on the other hand is more flexible in design, but cannot represent a smoothly curved surface without increasing the resolution by a sufficient number of creases. Such creases form a large number of vertices where the material largely deforms in-plane. Here, a curved fold forms a variety of surfaces using mostly separated small number of creases. Design examples of curved folding used for forming 3D surface are shown in Section 3.

Our goal is to find out further different types of applications of curved folding and to make curved folding applicable to a wider context of design by understanding its form variations and the geometry behind it. In this sense, curved folding is a relatively underexplored topic. Therefore, we start from introducing previous works by artists and designers and the applied geometric approach to analyze and design curved foldings. Here are the main contributions of our paper.

*We introduce the works by a variety of artists and designers who have deeply explored the forms of curved folding.*

*We show examples of curved folding used for the design of products and interior fixtures and the design procedure adopted.*

*We review successful geometric analysis of curved folding and the design methods based on geometric and computational means.*

## 2. CURVED CREASES IN ART AND DESIGN

2.1 Napkin folding

When we consider examples of curved creases, the trajectories of art, mathematics and education cross and we can observe concurrent developments in all fields. Napkin folding is surprisingly well documented in German since the 17th century, but not as art, rather as a teaching document. This decorative art with its complicated table decorations required manuals to teach all techniques. An early account of such a handbook with curved creases is the Trincir-Buch by Georg Philipp Harsdoerffer from 1652 (Sallas, 2010).

## 2.2 The Bauhaus model

The teaching work by Josef Albers at the first Bauhaus in 1927 and 1928 is documented in photographs and represents the first account of a specific curved crease model, which he taught in his 'Vorkurs', an introductory design class (Adler, 2004). Albers points out that working with materials and exploiting its properties leads to an efficiency of means as a material will be utilized to its maximum potential (Londenberg, 1963). The model is made of concentric circles with alternating mountain and valley folds seen in Figure 1 on the right and automatically folds into the shape seen on the left (Wingler, 1978). The crease pattern has different symmetries than the folded configuration.

A variation of the design was recreated by Irene Schawinsky, the wife of Alexander 'Xanti' Schawinsky, who was a Bauhaus student and later taught at Black Mountain College during the time Albers was teaching there. Her model shows a variation with a large hole in the center (McPharlin, 1944). Thoki Yenn publicized his version of the model in the 1980s, which he called 'Before the Big Bang' (Yenn, 1980's) and Kunihiko Kasahara, who learned of the model from Yenn, made many versions of it in his 'Extreme Origami' in 2003 (Kasahara, 2003). Erik and Martin Demaine started to explore this model in 1989 and made variations of it since. The model shown in Figure 2 at on the left uses multiple discs of paper that are joined together. The design on the right is comprised of multipl eindividual modules (Demaine et al., 2009). The sculptures are part of the Museum of Modern Art (MoMA) permanent collection. In 2008 Duks Koschitz togther with the Demaines created variations of this model that are based on crease patterns that use conic sections. The resulting shapes display very different symmetries than the symmetries of the crease pattern in Figure 3 (Koschitz et al., 2008).
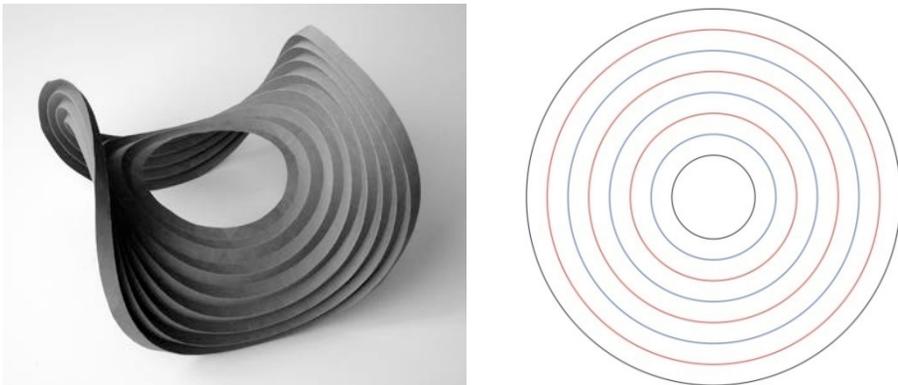


**Figure 1**: The Bauhaus design and its crease pattern.

**Figure 2:** Design by Erik Demaine and Martin Demaine, three joined discs and several modules
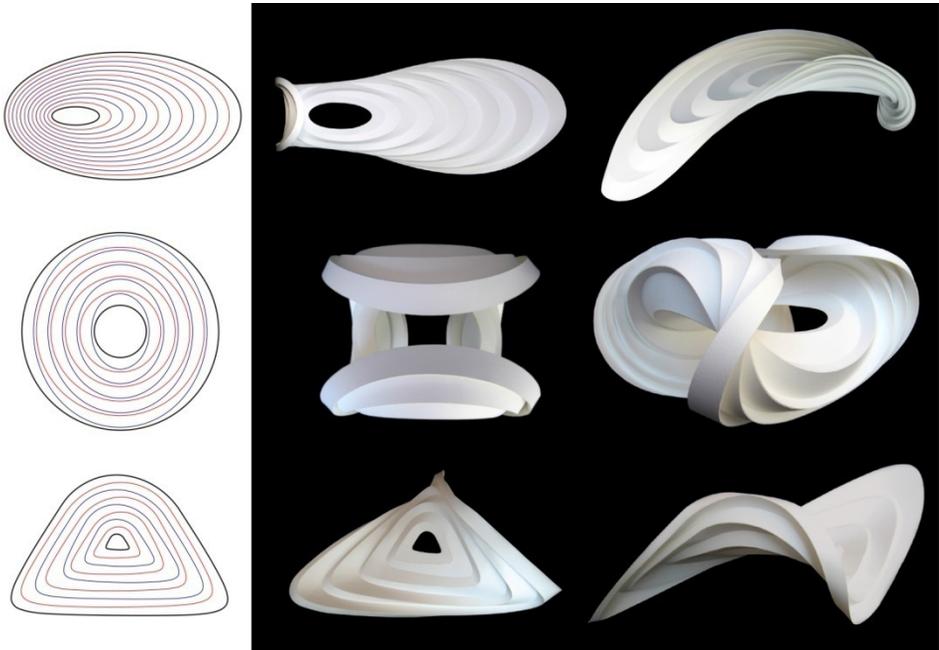


**Figure 3:** Variations of Bauhaus model by Duks Koschitz:

ellipses with shifted foci,

ellipses & circles,

quadratic splines with 2 parabolas & 1 circle

**2.3 Books on paper folding with curved creases of the 1970's**

Less known for his paper foldings but certainly recognized for his bookmaking art, Kurt Londenberg (1914-1995) published 'Papier und Form' featuring works of paper folding in 1972 with several new editions later on. He presents paper folding in various contexts including a section called 'architectural folding', in which he published the design in Figure 4. The design has similarities to the 'Groin Vault Twist' by contemporary folder Philip Chapman-Bell.

Many of the photographed models were made specifically for the book and he saw this publication as an educational contribution (Londenberg, 1963). Londenberg attributes great significance to Bauhaus educator and artist, Josef Albers, and reprinted his article on working with paper. In the same year Hiroshi Ogawa (Ogawa , 1972) published crease patterns in his 'Forms of Paper' and both authors should be considered as designers since they made their geometrically repeating sculptures themselves. Their works display artistic qualities beyond the didactic role they played in their book.
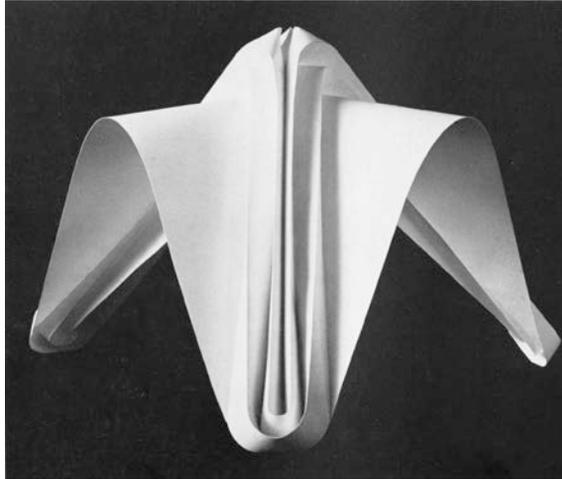


**Figure 4**: Design by Kurt Londenberg

**2.4 Two important figures of the 1970s**

The most expressive work from the 1970s that is not related to the Bauhaus model and uses curved creases has to be attributed to computer scientist David Huffman and artist Ron Resch. The contemporaries knew one another and had many discussions about

folding paper. Huffman remained true to his roots and took an analytical approach, while Resch was more interested in applied techniques for sculptures and other artistic endeavors. Both published and had a strong connection to computational processes, but only Resch used computers to realize some of his sculptural work such as the design in in Figure 3 at the top (Resch et al., 1974). The design is based on a tiling of a 3d curve which can also be seen in Figure 8 at the top. Resch, being concerned with fabrication methods and the expressive nature of art created this sculpture with an elaborate boundary that visually alters the tile from Figure 8.

Huffman's passion beyond his academic work was rooted in paper folding. He focused on tessellations with straight creases early on and it is hard to estimate when exactly he discovered curved creases for himself. Huffman owned the book by Hiroshi Ogawa with its curved crease patterns and several examples in this publication are comparable to Huffman's own investigations. It is however unclear when exactly he acquired the book. Ogawa's sculptures are fairly regular and his goal was to cover many techniques (Ogawa , 1972), however Huffman's investigations are far more rigorous.

Huffman's work can be described as true to the 'one piece of paper, no cuts' rule of folding purists as seen in Figure 3 at the bottom (Koschitz, 2014). Huffman may have not considered himself to be an artist, but his work is highly valued in the folding community both as artistic artifacts and mathematical investigations. He mentioned in a description of himself while teaching at UCSC. 'I don't claim to be an artist. I'm not even sure how to define art. But I find it natural that the elegant mathematical theorems associated with paper surfaces should lead to visual elegance as well' (Wertheim, 2004).

The shown example in Figure 5 at the bottom consists of six pairs of two different spirals arranged in a rotational tiling around the center of the crease pattern. Huffman created several dozen of these designs around 1978.

Both Resch and Huffman made their models with thin sheets of vinyl and pre-creased the folds. Resch used early CNC plotters at the University of Utah in the late 1960's and Huffman made his models by hand with a ball burnisher of ballpoint pen. Uncalendered PVC has a very similar appearance to white paper. The work of both designers is almost exclusively symmetric, often as regular and sometimes as rotational tilings.
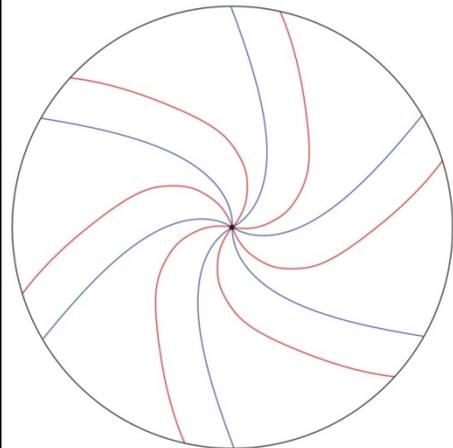
**Figure 5**: 'Space curve' by Ron Resch, Rotational tiling by David Huffman and crease pattern (ca.1978)

## 2.5 Contemporary art

Among the many contemporary folders and origami experts we would like to mention a few who have discussed their design approaches. Matthew Shilan for example states that he works with what he calls 'systems' and that he does not know what the result will be. Once a system of folding is initiated, the outcome is unknown, led as it is by the qualities of the material. According to him the process consists of 'exploration and

invention' (Smith, 2009). Saadya Sternberg studies geometric tessellations and creates a catalog of spiral tessellations that elaborates on techniques of how to use curved creases. He also recreated a Huffman model known as 'hexagonal column with cusps' (Sternberg, 2009). Jun Mitani, prolific folder and computer scientist, has created many designs with curved creases. The 'square based bud' model by Jeannine Mosely also ought to be mentioned here. It may seem to be necessary to discuss Paul Jackson's work in this context, specifically his 'one fold models', but when looked at closely it becomes evident that while the paper is curved the creases are straight lines (Thomas et al., 2001). The resulting sculptures are obviously developable, but do not use curved creases as the previously mentioned examples do.

Contemporary examples of paper foldings in Figure 6 by Robert Sweeney, Yuko Nishimura and T. Roy Iwaki continue to intrigue art focused audiences and display how regular repeated shapes have been used in art. Sweeney systematically tackles certain configurations and focuses on creating free standing or suspended objects often made of many pieces (Schmidt et al., 2009) and in some case at large scales. Nishimura on the other hand has a connection to her art through folding day to day commodities from kimonos to wrapping goods. She does not focus on any specific area of origami tessellations and is interested in expressing the Japanese soul through form (Smith, 2009). The shown example consists of a long sheet of paper that is arranged in a rotational configuration similar to a paper fan. T. Roy Iwaki created elaborate origami masks of animals and based his designs on simple basic shapes that he then used in complex aggregations. He was able to achieve the necessary concave and convex portions of a horse's head for example in Figure 6 on the right (Iwaki, 2010).
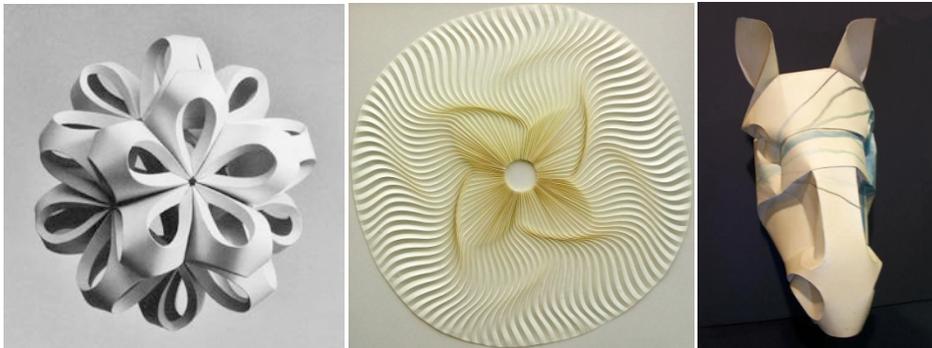


**Figure 6**: 'Modular sculpture' by Richard Sweeney (photo by the artist) (left), a 'relief' by Yuko Nishimura (photo by Yosuke Otomo) (center), 'Horse head' by T.Roy Iwaki (photo by Robert Lang) (right).

# 3. DESIGN IMPLEMENTATIONS IN PRODUCT AND INTERIOR DESIGN

Industrial designers, product designers and interior architects sometimes privilege practical approaches over artistic methods and since we know little about this geometry designer's opinions and descriptions of their own work can enlighten us in terms of how to design with something we don't fully understand. The choice of working with a specific material for a design project is often important to a designer, but we will focus on formal aspects of the work.

## 3.1 Lamp Designs by Poul Christiansen

The lamp designs by the LeKlint company are still produced by hand toady. The designs had been developed in the family and it is unclear when exactly the first examples of curved creases were created. Many designs consist of cylindrical configurations made of long narrow plastic sheets such as the example in Figure 7 (Jacobsen, 2008). The company hired Poul Christiansen in the 1970's to create designs with regular tilings. It is difficult to obtain crease patterns as the work is owned by the LeKlint Company and the work is now proprietary.



**Figure 7**: 'Pendant 172' by Poul Cristiansen.

## 3.2 Bench Design by Tim Herok, Markus Schein

The 'Liegengenerator project by Tim Herok and Markus Schein helps generate bench designs. The process starts with defining tight constraints for two edges of the bench.

One edge touches the ground and the other is the center line in the symmetry plane of the seating area (Trebbi, 2008). Schein set up a digital model that is using a genetic algorithm (Schein, 2002), which is looking for a solution that a user customizes by tweaking height and undulation parameters. The resulting plan spline represents the outline on the floor and is used to construct the section spline. After intersection points are plotted the surface can be constructed by lofting the discrete parts together. This design approach is different from previous ones as it places the designer further in the background. User defined parameters generate the final shape that was selected by an algorithm.

### 3.3 Metal Column Covers by Haresh Lalvani

Haresh Lalvani's work on column covers (Lalvani, 2003) is made with a very similar approach that uses a genetic algorithm to exhaustively explore a simple setup. The expressiveness of their 'orchestrated random' designs is remarkable when one takes the constraints into consideration. The column covers are part of the MoMA collection and are made in steel.

### 3.4 Car Design by Gregory Epps

Gregory Epps designed the car in Figure 8 as proof of concept prior to founding is company RoboFold. The resulting shapes of his method that starts by crumpling paper are irregular and have great expressive potential. Epp's playful design method rarely produces cylindrical results and most of his work focuses on regular tilings made in aluminium for large installations.
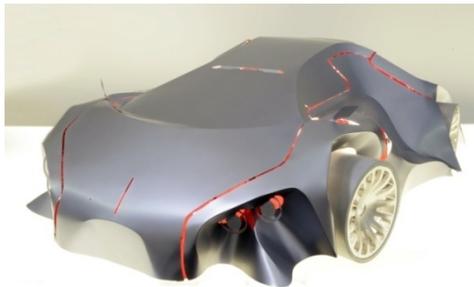


**Figure 8**: Car design by Gregory Epps (photo by the artist).

## 3.5 Deployable Structure by Curved Folds

Tachi designed a deployable cellular and tubular structure by assembling curved folded sheets (Tachi, 2013). The 1-DOF mechanism of curved surfaces is obtained through refinement of rigid origami structures. Unlike ordinary rigid origami, curved folds use the bending of the surface for deployment. Even though each sheet tends to be very flexible and floppy, the assembled structure becomes stiff and self-supports when deployed. Figure 9 shows a vault with 18 sheets sewn together along curved creases.



**Figure 9**: Tomohiro Tachi, Deployable Structure by assembly of curved foldings.

In concluding the industrial and interior design section we would like to point out a curious characteristic common to all mentioned examples, which is their bias towards symmetry. We believe this to be a result of functional requirements and material constraints. The difficulties designers face when working with curved creases can be addressed by exploiting the economy of means of symmetric designs.

# 4. MATHEMATICAL ANALYSIS AND COMPUTATIONAL METHODS

In order to make curved paperfolding applicable to design, it is necessary to understand its geometry. Curved creases are mathematically underexplored and no general representation exists today. We present some of the major approaches to the problem.

Origami, including curved folding, can be usually modeled as an intrinsically isometric embedding of a plane to a 3D Euclidean space. This definition is not enough, though, as the Nash–Kuiper theorem concludes that there always exists such an embedding in class $C^1$ (uncreased "folding") which can be arbitrarily close to a given double-curved surface. An example of this counterintuitive theorem (Nash, 1954) (Kuiper, 1955) involves non $C^2$ points densely covering the surface. In order to capture the actual behavior of paper folding, we define an origami surface as a $C^0$ surface which is piecewise $C^2$. The points at which the surface is not $C^1$ are called crease points; assuming the surface is piecewise-$C^2$, the crease points decompose into locally 1D curves.

## 4.1 Differential Geometric Analysis

The most fundamental result starts with a differential geometric approach, i.e., understanding the local behavior of the surface. Huffman (Huffman, 1976) describes the local behavior of a crease by introducing spherical trigonometry on the Gauss sphere, and his publication still remains a main reference today. Resch also investigated curved creases and stipulated that every space curve can be used to construct three distinct curved creases. He demonstrated his approach using early computer graphics rendering in Figure 10 (Resch et al., 1974) (Resch, 1974).
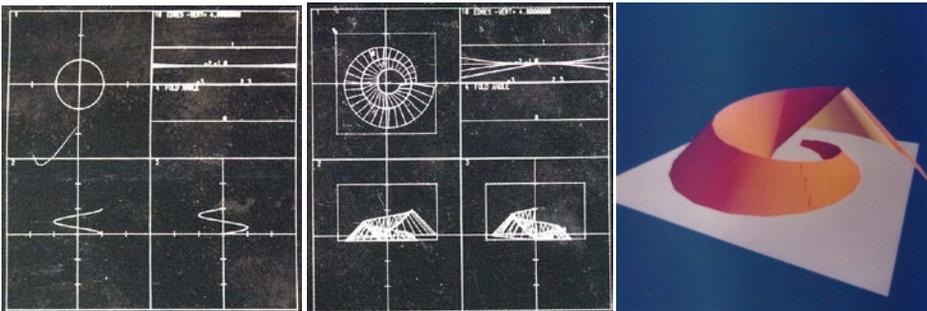


**Figure 10**: 'Space Curve' byRon Resch and Ephraim Cohen, screen shots of curve and surfaces, rendering.

Fuchs and Tabachnikov (Fuchs et al., 1999) further Huffman's work and contributed significantly to the understanding of developable surfaces and curved creases.. They assess that it is possible to fold an arbitrary curve drawn on paper into a 3D crease with higher curvature. If the curve is strictly convex and closes onto itself (e.g., circle) then the folded 3D crease is not in a plane. They also elaborate on the behavior of rulings along folded crease

Even though these local analyses form the base of other geometric design approaches, these general results themselves stop at the first crease, while multiple creases are applied in practical designs. Demaine et.al. (Demaine et al., 2014) investigate curved creases and surface between multiple curved creases, and give qualitative properties that a curved folding must satisfy. This characterization first succeeded in analyzing paper with non-trivial multiple creases (not planar), e.g., Lens tessellation designed by David Huffman in Figure 11. On the other hand, Demaine et. al. (Demaine et al., 2009b) describe how paper behaves between straight creases and mathematically answer why only curved creases can produce interesting curved surface. In other words, the surface surrounded by straight creases cannot bend and must stay polyhedral.
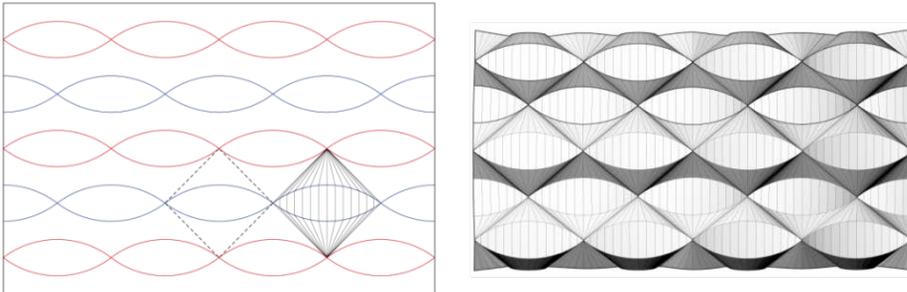


**Figure 11**: David Huffman, Lens Tesselations, reconstruction by Duks Koschitz.

## 4.2 Constructive Geometric Approach

One of the simplest design methods of curved folding with more than one crease is to use reflection. We start from a single developable surface and cut and reflect it by planes. The reflection is a special case of curved folding where the crease lies on a single osculating plane as described in Huffman's Primer on Paper (Huffman, 1976). This is a simple yet effective method and has been deployed by many artists. For example, Huffman's cone model is created using a single cone and its mirror reflections. The reconstruction process shows how Huffman designed these types of models (Demaine et al., 2010). Further reflections in Huffman's own archive that are

based on cylinders and cones can be reconstructed and analyzed in simple digital ways (Koschitz, 2014). Jun Mitani's software Ori-Ref allows users to these manipulations in the computer.

## 4.3 Inverse Calculation of a Crease

More advanced methods solve an inverse problem to connect known elements such as cones and cylinders with a curved crease. Geretschläger (Geretschläger, 2009) sets out to understanding curved creases by predefining the geometry of a piece of paper in a curved state. He then assumes the path of a crease and calculates the position of the part of the paper on the other side of the fold.

This type of inverse approach is useful to construct reusable modules for constructing symmetrically aligned or tessellated figures. Mosely analyzes curved creases of her own 'cube shape', a volumetric model she invented that uses several tiles of a simple crease pattern made of four semicircles (Mosely, 2002). Also, in her tessellation works, a curve is numerically calculated so that cones and cylinders symmetrically tessellate a plane (Mosely J. (2008).

## 4.4 Discrete Geometric Approach

In order to deal with fully generalized curved folding without predetermined assumptions on the form of surfaces, we need to globally solve geometric problems by discretizing and globally solving the geometric problems.

Lalvani uses genetic algorithms that select mutated straight polyhedra. His "Morphological Genome Project" is based on defining parameter sets or genes, which are then used to modify a polyhedral shape. The final selected results by the genetic algorithm are developable (Lalvani, 2003). The practical application of this work lead to metal column covers for interiors as described in Section 3.3.

The symmetric basic shape of the Spidron pattern can be considered as a discrete version of curved folding that follows a logarithmic curve (Erdély, 1979). The discretization is based on triangular strips to describe the developable surface. As triangulated strips can use its non-smooth nature to represent warped surfaces and this approximation can be problematic we should instead consider a planar quad strips as

Kilian and his colleagues or apply torsion-free constraints as Kergosien as his colleagues for obtaining ruling configurations.

Kergosien et al. take an engineering approach to investigate early simulations of paper (Kergosien et al., 1994). Starting from a generic curve they are able to fit a developable surface. If the boundary curve creates crossing rule lines their algorithm finds a curved crease within the boundary as seen in.

Kilian et al. model curved folding using planar quadrangle meshes (PQ-meshes) and deploy an optimization based method (Kilian et al., 2008). A case study they investigated is a car design by Gregory Epps that is made of a single piece of paper. The physical model is 3D scanned and an elaborate process of analysis, rule line searching, plane fitting and edge optimization follows that results in a description of the piecewise developable surface. The work can post-rationalize a scanned paper model, which is useful for fabrication for instance, but does not describe the folding process or generate novel forms, which still presents the main challenge today.

## 5. CONCLUSION

We have reviewed a small portion of previous works of curved folding in art and design, examples of industrial applications of curved folding, and mathematical and computational considerations reflected in some of the designs. We hope that this study helps the development of novel curved folding design in a structural context.

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